Optimal transport — classical and a little bit of quantum

Abstract of the introductory talk by Tamás Titkos. The purpose of this introductiory lecture is to provide a very gentle introduction to optimal transport. The original transport problem goes back to Gaspard Monge (∼1781), with significant advancements by Leonid Kantorovich. We mention that Kantorovich even received a Nobel Prize in economics (1975, shared with Tjalling Koopmans) for his contributions to the theory of optimum allocation of resources.

In the last decades, techniques using the theory of optimal transportation have achieved great success in several important fields of pure mathematics including probability theory, theory of partial differential equations, and geometry of metric spaces. The aim of this lecture is to explain both the Monge and the Kantorovich formulation of the transport problem, and to introduce the notion of the Wasserstein metric, a metric whose geometric characteristics have given a momentum for research in many areas of applied sciences like image processing, medical imaging, inverse imaging problems, and machine learning.

Abstract of the main lecture by Dániel Virosztek. The bulk of this lecture is devoted to discussing a carefully selected collection of the most instructive results in classical optimal transport theory. This collection includes

- various characterizations of optimal transport strategies via cyclical monotonicity and subdifferentials of convex functions,
- the Kantorovich-Rubinstein duality, along with its picturesque economic interpretation,
- basics of the kinetic/dynamical theory: displacement interpolation (Benamou-Brenier formula), connections to fluid mechanics.

In the remaining time, we will sketch the core ideas of some highly non-classical results of quantum optimal transport, such as the 1-1 correspondence of quantum channels and transport plans or the "quantum optimal transport is cheaper" phenomenon.